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INFERENCE ON QUANTILE REGRESSION PROCESS, AN ALTERNATIVE

Victor Chernozhukov

Working Paper 02-12
February 2002

Room E52-251
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INFERENCE ON QUANTILE REGRESSION PROCESS, AN ALTERNATIVE

VICTOR CHERNOZHUKOV

Abstract. A very simple and practical resampling test is offered as an alternative to inference based on Kmaladzation, as developed in in Koenker and Xiao (2002a). This alternative has competitive or better power, accurate size, and does not require estimation of non-parametric sparsity and score functions. It applies not only to iid but also time series data. Computational experiments and an empirical example that re-examines the effect of re-employment bonus on the unemployment duration support this approach.

Key Words: bootstrap, subsampling, quantile regression, quantile regression process, Kolmogorov-Smirnov test, unemployment duration

1. Introduction

Inference in quantile regression models, pioneered by the classical work of Koenker and Bassett (1978), is crucial to a wide range of economic analyses. For example, evaluation of the distributional consequences of social programs requires inference concerning nature, direction, and quantity of the impact throughout the entire outcomes distribution. See e.g. Abadie (2002), Buchinsky (1994), Heckman and Smith (1997), Gutenbrunner and Jurečková (1992), McFadden (1989), Koenker and Xiao (2002a), and Portnoy (2001). Just like in the classical p-sample theory, e.g. Doksum (1974) and Shorack and Wellner (1986), this kind of inference is based on the *empirical quantile regression process*. It differs however from the early approaches by replacing the basic (indicator) regressors with general ones.

The main difficulty associated with such inference is the *Durbin problem* – the model's features, estimated nuisance parameter, or non-i.i.d. data induce parameter-dependent asymptotics, jeopardizing distribution-free inference.[1] In a recent Econometrica paper Koenker and Xiao (2001) proposed an ingenious and intricate theory, based on Khmaladze transformation, that purges the tests statistics from the non-distribution-free components, restoring

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This project would have been possible without receiving help and encouragement from Takeshi Amemiya and Roger Koenker. I would like to express my appreciation and gratitude to them. I also thank Christian Hansen for very useful comments.

[1]The problem gained importance after a long series of remarkable works by Durbin, listed in the bibliography.

distribution-free inference. The approach uses recursive projections to annihilate the predictable component in the process, leaving a martingale that limits to a standard Brownian motion, thus overcoming the Durbin problem.

Here we suggest a simple resampling alternative that (i) does not require the somewhat complex Khmaladze transformation, (ii) does not require the estimation of the nonparametric nuisance and score functions, (iii) has an accurate size and the optimal power (in the sense that its has same power as the test with known critical value[2]), which makes it very competitive with Khamaldzation, (iv) is robust to dependent data, and (v) is computationally and practically attractive. Therefore, the approach is a useful complement to Khmaladzation and is aimed at substantively expanding the scope of empirical inference.

The basic idea is extremely simple. The key statistic is based on the quantile regression process. The statistic has a limit distribution, denoted H, under the null hypothesis. In order to estimate H correctly, regardless of whether the null is true or not, we resample an appropriately re-centered quantile process. As a result, H as well as the entire null law of the process are correctly estimated under local departures from the null. For resampling purposes, we choose the subsample bootstrap, cf. Politis, Romano, and Wolf (1999), which has computational, practical, and certain theoretical advantages over the usual (unsmoothed) bootstrap for quantile regression, cf. Buchinsky (1995) and Sakov and Bickel (2000). Horowitz (1992)'s smoothed bootstrap may also be an attractive resampling mechanism for these tests.

The underlying principle differs from the *conventional* bootstrap tests for goodness of fit. The conventional tests resample from a probability model that is consistent with the null, see e.g Romano (1988), Andrews (1997), and Abadie (2002). Although such approach is potentially useful in quantile regression settings, its validity remains unknown, because the quantile regression families estimated in emprical work are typically mis-specified and incomplete probability models (regression quantile lines are not constrained to avoid crossing, and the tail regression quantiles are not estimated).

In what follows, we use P_n^* to denote (outer) probability, which possibly depends on n, and $\Rightarrow$ and $\xrightarrow{d}$ to denote weak convergence in a space of bounded functions and convergence in distribution for random vectors, respectively, under P_n^*.

2. The Testing Problem

The questions posed in the fundamental econometric and statistical literature are whether the treatment exhorts a pure location effect, a location-scale effect, or a general shape effect, cf. Doksum (1974), Koenker and Machado (1999), Koenker and Xiao (2002a), or, for example, a stochastic dominance effect, cf. Abadie (2002), Heckman and Smith (1997), and McFadden (1989). Quantile regression is an important and practical tool for learning about such distributional phenomena.

[2]Khmaladze tests do not generally have this property, see Koenker and Xiao.

Suppose Y is the outcome variable, and X are regressors. Let $F_{Y|X}$ and $F^{-1}_{Y|X}(\tau)$ denote the conditional distribution function and the τ-quantile of Y given X. The basic conditional quantile model takes the linear in parameters form:

$$F^{-1}_{Y|X}(\tau) = X'\beta_n(\tau),$$

for all $\tau \in \mathcal{T}$, where $\mathcal{T} \equiv [\epsilon, 1-\epsilon]$ are quantiles of interest. This is a random coefficient model $Y = X'\beta_n(U)$, where $U \sim U(0,1)$. The stated model allows regressors to affect the entire shape of the conditional distribution and includes the classical linear location and location-scale models as special cases. To facilitate the local power analysis, parameter $\beta_n(\tau)$ is made dependent on the sample size n.

As in Koenker and Xiao (2002a), we consider the following null hypothesis:

$$R(\tau)\beta_n(\tau) - r(\tau) = \Psi(\tau), \quad \tau \in \mathcal{T}, \tag{1}$$

where $R(\tau)$ denotes a $q \times p$ matrix, $q \leq p = \dim(\beta)$, $r \in \mathbb{R}^q$, and $\Psi(\tau)$ denotes a known function $\Psi : \mathcal{T} \to \mathbb{R}^q$. We assume that functions $R(\tau)$, $\Psi(\tau)$, $r(\tau)$, and $\beta_0(\tau) \equiv \lim_n \beta_n(\tau)$ are continuous in τ.

The tests will be based on the Koenker-Bassett quantile regression process $\widehat{\beta}_n(\cdot)$:

$$\widehat{\beta}_n(\tau) = \arg\min_{\beta \in \mathbb{R}^d} \sum_{t=1}^{n} \rho_\tau\left(Y_i - X_i\beta\right), \quad \tau \in \mathcal{T},$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$. Other estimators, such as Chamberlain's minimum distance or instrumental variable quantile regression estimators, can be considered as well, depending on the problem. We will focus on the basic inference process:

$$v_n(\tau) = \left(\widehat{R}(\tau)\widehat{\beta}_n(\tau) - \widehat{r}(\tau) - \Psi(\tau)\right), \tag{2}$$

and derived from it Kolmogorov-Smirnov and Smirnov statistics $S_n = f(\sqrt{n}v_n(\cdot))$,

$$S_n = \sqrt{n}\sup_{\tau \in \mathcal{T}} \|v_n(\tau)\|_{\widehat{V}(\tau)}, \quad S_n = n\int_{\mathcal{T}} \|v_n(\tau)\|^2_{\widehat{V}(\tau)} d\tau, \tag{3}$$

where $\|a\|_V \equiv \sqrt{a'Va}$, the symmetric $\widehat{V}(\tau) \xrightarrow{p_n} V(\tau)$ uniformly in τ, and $V(\tau)$ is a positive definite symmetric matrix uniformly in τ. The choice of V and $\widehat{V}$ is discussed in section 3.

Example 1 (***Location Hypothesis***). An important hypothesis is that of the classical *location-shift* regression

$$F^{-1}_{Y|X}(\tau) = X'\alpha + \gamma \cdot F_V^{-1}(\tau), \quad \text{or} \quad Y = X'\alpha + \gamma \cdot V,$$

where V is independent of X. In this case, $R(\tau) = R = [0 : I_{p-1}]$ and $r(\tau) = r = (\alpha_2, ..., \alpha_p)'$, which asserts that the quantile regression slopes are constant, independent of τ. The component r can be estimated by any method consistent with the null, for example LAD, OLS, and others in Koenker and Xiao (2002a).

Example 2 (***Location-scale Hypothesis***). The *location-scale shift* regression constraints X to affect only the location and scale of Y, but not any other moments:

$$F^{-1}_{Y|X}(\tau) = X'\alpha + X'\gamma \cdot F^{-1}_V(\tau), \quad \text{or} \quad Y = X'\alpha + X'\gamma \cdot V,$$

where V is independent of X. In this case, $r(\tau) = \alpha + \gamma \cdot F^{-1}_V(\tau)$, $R = [0 : I_{p-1}]$, and $\Psi(\tau) = 0$. Estimates of α and $\gamma \cdot F^{-1}_V(\tau)$ can be obtained by using OLS projection of each component of slopes vector $\widehat{\beta}_{-1}(\cdot)$ on the intercept $\widehat{\beta}_1(\cdot)$, see Koenker and Xiao (2002a).

Example 3 (***Stochastic Dominance Hypothesis***). Suppose $D = 1$ denotes receipt and $D = 0$ denotes non-receipt of a treatment. The test of *stochastic dominance*, or whether the treatment is unambigiously beneficial, in the model

$$F^{-1}_{Y|D,X}(\tau) = D\delta(\tau) + X'\theta(\tau),$$

involves the dominance null

$$\delta(\tau) \geq 0, \text{ for all } \tau \in \mathcal{T}$$

versus the non-dominance alternative

$$\delta(\tau) < 0, \text{ for some } \tau \in \mathcal{T}.$$

In this case, the least favorable null involves $r(\tau) = 0$, $R = -[1, 0...]$, $\Psi(\tau) = 0$, and $\beta(\tau) = (\delta(\tau), \theta(\tau)')'$, and one may use the one one-sided Kolmogorov-Smirnov or Smirnov statistics $S_n = \sqrt{n}\inf_{\tau\in\mathcal{T}} \max(-\widehat{\delta}_n(\tau), 0)$ and $S_n = \sqrt{n}\int_{\mathcal{T}} \|\max(-\widehat{\delta}_n(\tau), 0)\|^2_{V(\tau)} d\tau$ to test the hypothesis.

We will maintain the following assumptions.

A.1 $(Y_t, X_t, t \leq n)$ is stationary and strongly mixing on probability space $(\Omega, \mathcal{F}, P_n)$.

A.2 Law of $(Y_t, X_t, t \leq n)$, $P^{[n]}_n$, is contigious to some $P^{[n]}$,[3] and either
(a) for a fixed continuous function $p(\tau) : \mathcal{T} \to \mathbb{R}^q$ and for each n

$$R(\tau)\beta_n(\tau) - r(\tau) = \Psi(\tau) + g(\tau), \quad g(\tau) = p(\tau)/\sqrt{n}, \text{ or,}$$

(b) for a fixed continuous function $g(\tau) : \mathcal{T} \to \mathbb{R}^q$ and for each n

$$R(\tau)\beta(\tau) - r(\tau) = \Psi(\tau) + g(\tau).$$

A.3 (a) Under any local alternative, A2(a), $\sqrt{n}\left(\widehat{\beta}_n(\cdot) - \beta_n(\cdot)\right) \Rightarrow b(\cdot)$, $\sqrt{n}(\widehat{R}(\cdot) - R(\cdot)) \Rightarrow \rho(\cdot)$, $\sqrt{n}\,(\widehat{r}(\cdot) - r(\cdot)) \Rightarrow -\varsigma(\cdot)$, jointly in $\ell^\infty(\mathcal{T})$, where (b, ρ, ς) are jointly zero mean Gaussian functions with nondegenerate covariance kernel.
(b) Under the global alternative, A2(b), the same holds, except that the limit $(\tilde{b}, \tilde{\rho}, \tilde{\varsigma})$ needs not have the same distribution as in A3(a).

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[3]As defined e.g. on p. 87 in van der Vaart (1998)

A.1 allows a wide variety of data processes: iid, time series, and panels. Mixing is sufficient but is not necessary for consistency of subsampling. Stationarity can be replaced by more general stability conditions, see ch. 4 in Politis, Romano, and Wolf (1999). A.2(a) and A.2(b) formulate a local and a global alternative. A.3 is very general condition, that is implied by a wide variety of conditions in the literature, most remarkable and general of which are given in Portnoy (1991), who allows shape heteroscedasticity and dependent data. Thus A.3. substantively generalizes Koenker and Xiao's (2001) or Koenker and Machado's (1999) conditions (local-to-location-scale assumption and iid sampling) which elegantly suit the hypotheses in Examples 1 and 2, but are restrictive and not necessary in Example 3.

Proposition 1. *1. Under conditions A1, A2a, A3, in $\ell^\infty(\mathcal{T})$*

$$\sqrt{n}v_n(\cdot) \Rightarrow v(\cdot) \equiv \underbrace{u(\cdot) + d(\cdot)}_{v_0(\cdot)} + p(\cdot),$$

where $u(\tau) = R(\tau)'b(\tau)$ and $d(\tau) = (\beta_0(\tau)\rho(\tau) + \varsigma(\tau))$. Under the null, $p = 0$,

$$S_n \Rightarrow S \equiv f(v_0(\cdot)).$$

2. Under A1, A2b, A3, $\sqrt{n}(v_n(\cdot) - g(\cdot)) \Rightarrow \tilde{v}(\cdot) \equiv \tilde{u}(\cdot) + \tilde{d}(\cdot)$, where $\tilde{u}(\tau) = R(\tau)'\tilde{b}(\tau)$ and $\tilde{d}(\tau) = (\tilde{\beta}_0(\tau)\tilde{\rho}(\tau) + \tilde{\varsigma}(\tau))$. And $S_n \xrightarrow{p_{\tilde{n}}} \infty$ if $f(\sqrt{n}g(\cdot) + O_p(1)) \xrightarrow{p_{\tilde{n}}} \infty$ (which is true for statistics in (3) once $g \not\equiv 0$).

The limit consists of three components that illustrate the *Durbin problem*:

1. The *usual* component u is typically a Gaussian process with non-standard covariance kernel, so its distribution can not be feasibly simulated. This problem may be assumed away by imposing iid conditions. However, the problem does *not* go away, once the data is a time series or a panel. In such setting, Koenker and Xiao's method unfortunately does not apply in its present form.

2. Component d is the *Durbin* component that is present because R and r are estimated. Koenker and Xiao isolate d as a chief problem that makes the entire term v to have a nonstandard covariance kernel. They use Khmaladzation to annihilate this component.

3. Component p, which describes deviations from the null, determines the test's *power*. As Koenker and Xiao show, the Khmaladzation inadvertently removes some portion of this component as well. In fact they gave examples where p is removed completely, such as piece-wise linear densities. Such densities are not commonplace, yet they can easily approximate a well behaved density. Nevertheless, Koenker and Xiao show that Khmaladzation has respectable power in most *practical* cases.

Khmaladzation requires estimation of several nonparametric nuisance functions – the sparsity functions and various score functions, see Koenker and Xiao for details. The feasibility of this may depend on the underlying model. Under a location-scale shift model, the procedure is not laborious. Otherwise, for instance in Example 3, estimation of score functions is more difficult and has not been implemented nor had its theory been established.

In the next section, we describe a simple approach that is very useful in practice, does not erase components of p under any circumstances, and does not require nuisance function estimation. From a constructive point of view, the approach is not intended to be a critique of the Koenker and Xiao methods, which are brilliant and useful in many conceivable cases. Rather, the approach is meant to be a useful complement, aimed at substantively expanding the scope of empirical inference.

3. Resampling Test and Its Implementation

3.1. **The Test.** The approach is based on the *mimicking process* $\bar{v}$ and *statistic* $\mathcal{S}_n$:

$$\bar{v}_n(\tau) = v_n(\tau) - g(\tau), \quad \mathcal{S}_n = f(\bar{v}_n(\cdot)).$$

Proposition 2. *1. Given A.1, A2a, A.3* $\bar{v}_n(\cdot) \Rightarrow v_0(\cdot), \quad \mathcal{S}_n \Rightarrow S$. *2. Given A.1, A2b, A.3* $\bar{v}_n(\tau) \Rightarrow \bar{v}(\cdot) = \tilde{u}(\cdot) + \tilde{d}(\cdot), \quad \mathcal{S}_n \Rightarrow \mathcal{S} \equiv f(\bar{v}(\cdot))$.

Under local alternatives, the statistic $\mathcal{S}_n$ *correctly mimics the null behavior* of S_n, *even when the null is false.* This does not happen under global alternatives, but this is not important.

In what follows we use $v_n(\tau)$ itself to estimate $g(\tau)$, and use the subsample bootstrap to consistently estimate the distribution of $\mathcal{S}$, which equals that of S under the null hypothesis. The usual bootsrap will also work, but subsampling is preferable on both computational grounds explained in Buchinsky (1995) and theoretical reasons given in Sakov and Bickel (2000).[4]

The basic idea of the subsample bootstrap, introduced by Politis, Romano, and Wolf (1999), is to approximate the sampling distribution of a statistic based on the values of this statistic computed over smaller subsets of data. The resampling tests based on subsampling are done in three steps.

Step 1. For cases when $W_t = (Y_t, X_t)$ is iid, construct all subsets of size b. The number of such subsets B_n is "n choose b." For cases when $\{W_t\}$ is a time series, construct $B_n = n-b+1$ subsets of size b of the form $\{W_i, ..., W_{i+b-1}\}$. Compute the inference process $v_{b,n,i}(\cdot)$, for each i-th subset, $i \leq B_n$.[5]

Denote by v_n the inference process computed over the entire sample; and by $v_{b,n,i}$ the inference process computed over the i-th subset of data:

$$v_{b,n,i}(\tau) \equiv \left(\widehat{R}_{b,n,i}(\tau)\widehat{\beta}_{b,n,i}(\tau) - \widehat{r}_{b,n,i}(\tau) - \Psi(\tau)\right)$$

[4]Another attractive alternative is the smoothed bootstrap as in Horowitz (2001). Subsampling is used for pragmatic, computational reasons. In addition, Sakov and Bickel (2000) show that subsampling, combined with interpolation, yields the same minimax order of occupancy as smoothing once subsample size $b \propto n^{-2/5}$.

[5]A smaller number B_n of randomly chosen subsets can also be used, if $B_n \to \infty$ as $n \to \infty$, cf. Section 2.5 in Politis, Romano, and Wolf (1999).

and define $\mathcal{S}_{n,b,i} \equiv f(\sqrt{b}[v_{b,n,i}(\cdot) - v_n(\cdot)])$, for instance

$$\widehat{\mathcal{S}}_{n,b,i} \equiv \sup_{\tau \in \mathcal{T}} \sqrt{b} \| v_{n,b,i}(\tau) - v_n(\tau) \|_{\widehat{V}(\tau)} \text{ or } \widehat{\mathcal{S}}_{n,b,i} \equiv b \int_{\mathcal{T}} \| v_{n,b,i}(\tau) - v_n(\tau) \|^2_{\widehat{V}(\tau)} d\tau,$$

for cases when S_n is Kolomogorov-Smirnov or Smirnov statistics, respectively. Define

$$G(x) \equiv Pr\{\mathcal{S} \leq x\} \quad \text{and} \quad H(x) \equiv Pr\{S \leq x\}.$$

As $b/n \to 0$ and $b \to \infty$, $\sqrt{b}\|v_n(\cdot) - g(\cdot)\| = \sqrt{b} \times O_p(1/\sqrt{n}) \xrightarrow{P_n^*} 0$, including when $g(\tau) = p(\tau)/\sqrt{n}$. Therefore $\sqrt{b}\|v_{n,b,i}(\cdot) - g(\cdot) + (g(\cdot) - v_n(\cdot))\| = \sqrt{b}\|v_{n,b,i}(\cdot) - g(\cdot) + o_p(1)\|$, uniformly in i. Therefore, the distribution of $\widehat{\mathcal{S}}_{n,b,i}(\tau, g)$ can consistently estimate G, which *coincides with H under local alternatives.* Thus, the following steps are clear.
Step 2. Estimate $G(x)$ by

$$\widehat{G}_{n,b}(x) = B_n^{-1} \sum_{i=1}^{B_n} 1\{\widehat{\mathcal{S}}_{n,b,i}(\tau) \leq x\}.$$

Step 3. The critical value is obtained as the $1-\alpha$-th quantile of $\widehat{G}_{n,b}(\cdot)$:[6]

$$c_{n,b}(1-\alpha) = \widehat{G}^{-1}_{n,b}(1-\alpha).$$

The size α test rejects the null hypothesis when $S_n > c_{n,b}(1-\alpha)$.

Theorem 1. *Given A.1 - A.3 as $b/n \to 0, b \to \infty, n \to \infty$, $B_n \to \infty$,*

(i) *When the null is true, $p = 0$, if H is continuous at $H^{-1}(1-\alpha)$:*

$$c_{n,b}(1-\alpha) \xrightarrow{P_n} H^{-1}(1-\alpha), \quad P_n(S_n > c_{n,b}(1-\alpha)) \to \alpha.$$

(ii) *Under local alternative A2a, $p \not\equiv 0$, if H is continuous at $H^{-1}(1-\alpha)$:*

$$c_{n,b}(1-\alpha) \xrightarrow{P_n} H^{-1}(1-\alpha), \quad P_n(S_n > c_{n,b}(1-\alpha)) \to \beta,$$

where $\beta = Pr\left(f(v_0(\cdot) + p(\cdot)) > H^{-1}(1-\alpha)\right)$.

(iii) *Under global alternative A2b, if G is continuous at $G^{-1}(1-\alpha)$, and $S_n \xrightarrow{P_n^*} \infty$:*

$$c_{n,b}(1-\alpha) \xrightarrow{P_n} G^{-1}(1-\alpha), \quad P_n(S_n > c_{n,b}(1-\alpha)) \to 1.$$

(iv) *$H(x)$ and $G(x)$ are absolutely continuous at $x > 0$ when the covariance function of v and $\bar{v}$ is nondegenerate.*

Thus the resampling test is asymptotically unbiased and has the *same power* as the corresponding test that uses a known critical value. Furthermore if as $p(\tau) \to \infty$ or $-\infty$, $f(v_0(\cdot) + p(\cdot)) \to \infty$, the power β goes to one. Under global alternatives, the estimated critical values are $O_p(1)$ and $S_n \xrightarrow{P_n^*} \infty$ for Kolmogorov-Smirnov and Smirnov statistics.

[6]In practice it may be useful to account for an error in $\widehat{G}^{-1}_{n,b}(1-\alpha)$ caused by B_n being "small". In simulations, we added 1.69 times an estimate of standard error to $\widehat{G}^{-1}_{n,b}(1-\alpha)$, to make the test more conservative when B_n is small.

3.2. **Approximations.** It may be sometime more practical to use a grid $\mathcal{T}_n$ in place of $\mathcal{T}$ with the largest cell size $\delta_n \to 0$ as $n \to \infty$.

Corollary 1. *Propositions 1 and 2 and Theorems 1 and 2 are valid for piece-wise constant approximations of the finite-sample processes, given that $\delta_n \to 0$ as $n \to \infty$.*

3.3. **Estimation of $V(\tau)$.** In order to increase the test's power we could set

$$V(\tau) = V^*(\tau) = \operatorname{Var}[\bar{v}(\tau)]^{-1},$$

which is a (generalized) Andersen-Darling weight. In iid samples, there are many methods for estimating $V^*(\tau)$, *uniformly consistently in* τ. We are not aware of any results for more general cases. Note that we would need a Newey and West (1987) type estimator that is consistent uniformly in τ.

Subsampling itself can be used to estimate $V(\tau)$, even without assuming asymptotic integrability conditions. This is possible by using a percentile method in conjunction with asymptotic normality.

Consider the truncated variance

$$V_K^*(\tau) = \operatorname{Var}[\bar{v}_K(\tau)]^{-1}, \text{ where } \bar{v}_K(\tau) = \bar{v}(\tau) \times 1_{K(\tau)}(\bar{v}(\tau)),$$

$K(\tau) = \times_{j=1}^{p}[L_j(\tau), U_j(\tau)]$ is a large compact set. E.g., $L_j = \alpha$-quantile of $\bar{v}_j(\tau)$, and $U_j = 1 - \alpha$-quantile of $\bar{v}_j(\tau)$. We can estimate $V_K^*(\tau)$ using Theorem 2 stated below. Note that having estimated the truncated variance, we may stop there since for a large K, $V_K^*(\tau) \approx V^*(\tau)$, and Theorem 1 applies to any positive definite symmetric $V(\tau)$.

Second, using that $\bar{v}(\tau) \stackrel{d}{=} N(0, V^*(\tau))$, we can use the percentile method to obtain an estimate of diagonal elements of $V^*(\tau)$ based on $V_K^*(\tau)$. Using symmetrically trimmed correlations, we can then estimate off-diagonal elements. In simulations we simply used un-truncated variances.

Theorem 2 provides the uniformly consistent in τ estimates of any truncated moments of the process $\bar{v}_n(\tau)$, including the trimmed correlations. This theorem is a direct consequence of the ingenious results of Politis, Romano, and Wolf (1999).

Let $\tau \mapsto v(\tau)$ be an element of $\ell^\infty(\mathcal{T})$, equipped with the sup norm, and $\mathbf{L}(c, k)$ be a class of measurable Lipshitz functions $\varphi : \ell^\infty(\mathcal{T}) \to \mathbb{R}^K$ that satisfy:

$$\|\varphi(v) - \varphi(v')\| \le c \cdot \sup_\tau \|v(\tau) - v'(\tau)\|, \quad \|\varphi(v)\| \le k,$$

where c and k are suitably chosen positive constants. For probability laws Q and Q', define the bounded Lipshitz metric (which metrizes weak convergence) as

$$\rho_{BL}(Q, Q') = \sup_{\varphi \in \mathbf{L}} \|E_Q\varphi - E_{Q'}\varphi\|.$$

Useful examples of φ include $\varphi(\bar{v}) = \bar{v}(\tau)^m f_K(\bar{v}(\tau))$, where $(v_1, ..., v_p)^m \equiv v_1^{m_1} \times ... \times v_p^{m_p}$ and we replace the indicator $1_{K(\tau)}(\bar{v}(\tau))$ by a smooth approximation $f_{K(\tau)}(\bar{v}(\tau))$ which

vanishes outside compact set K, for all τ. This defines all kinds of truncated moments and correlations. For example, if $\bar{v}(\tau)$ is a scalar (for clarity sake), then

$$\widehat{\mathrm{Var}}[\bar{v}_K(\tau)] = E_{L_{n,b}}\left[\bar{v}(\tau)^2 f_K(\bar{v}(\tau))\right] = B_n^{-1}\sum_{i=1}^{B_n}\left[(v_{n,b,i}(\tau) - v_n(\tau))^2 f_K(v_{n,b,i}(\tau) - v_n(\tau))\right],$$

where $L_{n,b}$ denotes the subsampling (outer) law of $v_{n,b,i}(\cdot) - v_n(\cdot)$ in $\ell^\infty(\mathcal{T})$.

Theorem 2. *Under assumption A.1-A.3, letting* L *and* L_0 *denote the laws of* $\bar{v}(\cdot)$ *and* $v_0(\cdot)$ *in* $\ell^\infty(\mathcal{T})$, *respectively,*

$$\rho_{BL}(L_{n,b}, L) \xrightarrow{P_n^*} 0,$$

and L *equals* L_0 *under local alternatives. In particular, for functions* $(v \mapsto v(\tau)^m f_{K(\tau)}(v(\tau))$, $\tau \in \mathcal{T})$ *within* $\mathbf{L}(c, k)$,

$$\sup_{\tau\in\mathcal{T}}\left\| E_{L_{n,b}}\left[\bar{v}(\tau)^m f_{K(\tau)}(\bar{v}(\tau))\right] - E_L\left[\bar{v}(\tau)^m f_{K(\tau)}(\bar{v}(\tau))\right] \right\| \xrightarrow{P^*} 0.$$

The last statement remains true even when $f_{K(\tau)}(\cdot)$ *is replaced by* $1_{K(\tau)}(\cdot)$.

3.4. **Choice of Block Size.** In Sakov and Bickel (1999) and in Politis, Romano, and Wolf (1999) various rules are suggested for choosing appropriate subsample size. Politis, Romano, and Wolf (1999) focus on the calibration and minimum volatility methods. The calibration method involves picking the optimal block size and appropriate critical values on the basis of simulation experiments conducted with a model that approximates a situation at hand. The minimum volatility method involves picking (or combining) among the block sizes that yield more stable critical values. More detailed suggestions emerge from Sakov and Bickel (1999) and Buchinsky (1995). Sakov and Bickel (2000) suggest that choosing $b = kn^{2/5}$ yields[7] the optimal minimax accuracy (in conjunction with extrapolation). Our own experiments indicated that the constant k between 3 and 10 are attractive both computationally and qualitatively, which well accorded with the results of Sakov and Bickel (1999) for the sample median.

4. A Computational Example

The computational experiment that we consider is that of Koenker and Xiao (2002a). This allows us to compare the performance of the resampling test vs. Khamaladzation without prejudicing against the latter. Consider the location-shift hypothesis as in Example

[7]Their result is for the subsample bootstrap with replacement. However, the replacement and non-replacement versions are asymptotically equivalent once $b^2/n \to 0$. See e.g. Politis, Romano, and Wolf (1999).

1. The data is generated from the model:

$$\begin{aligned} Y_i &= \alpha + \beta X_i + \sigma(X_i) \cdot \epsilon_i, \\ \sigma(X_i) &= \gamma_0 + \gamma_1 \cdot X_i, \\ \epsilon_i &\sim N(0,1), \ X_i \sim N(0,1), \\ \alpha &= 0, \beta = 1, \gamma_0 = 1. \end{aligned}$$

Under the null hypothesis $\gamma_1 = 0$. We examine the empirical rejection probabilities for the test for different choices of sample sizes and heteroscedasticity parameter γ_1. In constructing the test, we used the OLS estimate of $\widehat{\beta}$ and $\mathcal{T} = [.05, .95]$. When $\gamma_1 = 0$ the model is a location-shift model, and the rejection rates yield the empirical sizes. When $\gamma_1 \neq 0$ the model is a heteroscedastic model, and the rejection rates give the empirical powers. Table 1 reports the results and compares them with Khmaladzation. Other details of the set up are as those reported in Koenker and Xiao (2002b).

Table 2 speaks for itself. The resampling test is powerful and accurate even in small samples. From these results, it is fair to say that the resampling test emerges as a respectable, serious complement to the Khamaladzation method. The method is also quite robust to variation of subsample size – a wide variety of subsample sizes performs very well (including when the resampling mechanism is the n out of n bootstrap), suggesting that even fairly small sub-samples ($b = 5n^{2/5}$) are both computationally and qualitatively attractive.

5. An Empirical Application

To illustrate the present approach, we will re-analyze and expand on the main empirical question considered in Koenker and Xiao (2002a). The question concerns the Pennsylvania re-employment bonus experiment conducted by the U.S. Department of Labor,[8] which was conducted in the 1980's in order to test the incentive effects of an alternative compensation scheme for unemployment insurance (UI). In these controlled experiments, UI claimants were *randomly* offered a cash bonus if they found a job within some prespecified of time and if the job was retained for a specified duration. The goal was to evaluate the impact of such a scheme on the unemployment duration.

As in Koenker and Xiao (2002a) we focus on the compensation schedule that includes a lump-sum payment of six times the weekly unemployment benefit for claimants establishing the reemployment within 12 weeks (in addition to the usual weekly benefits). The definition of unemployment spell includes one waiting week, with the maximum of uninterrupted full weekly benefits of 27.

The model under consideration is the linear conditional quantile model for the logarithm of duration:

$$Q_{\log(T)}(\tau|X) = \alpha(\tau) + \delta(\tau) \cdot D + X'\beta(\tau),$$

[8]There is a significant empirical literature focusing on the analysis of this and other similar experiments, see e.g. Meyer (1995)'s review.

where T is the duration of unemployment, D is the indicator of the bonus offer, and X is a set of socio-demographic characteristics (age, gender, number of dependents, location within the state, existence of recall expectations, and type of occupation). Further details are given in Koenker and Bilias (2001). The estimate of $\widehat{\delta}(\cdot)$ is plotted in Figure 1.

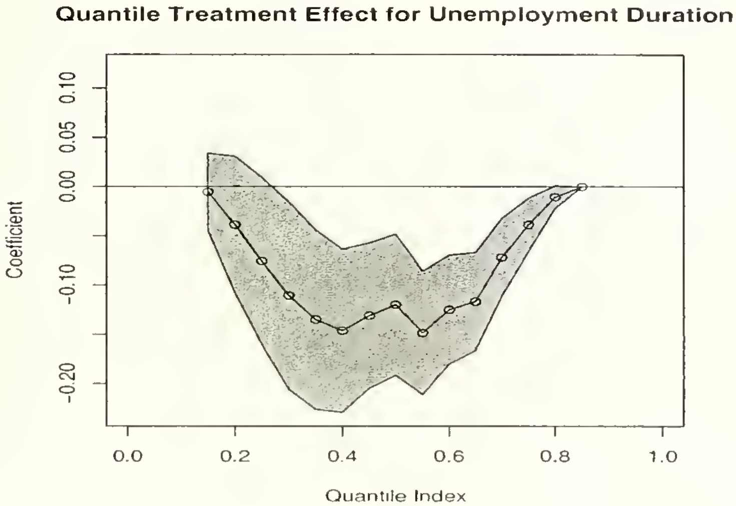

FIGURE 1

The three basic hypotheses described in table 3 include:

- treatment effect is constant across most of the distribution ($\mathcal{T} = [.15, .85]$),
- treatment affects only the location and scale of the outcome $\log(T)$,
- treatment effect is unambiguously beneficial: $\delta(\tau) \leq 0$ for all $\tau \in \mathcal{T}$.

These hypotheses specialize Examples 1-3 to the present case. The resampling test is implemented following section 3, and the results are given in Table 3. The tests were implemented for subsample size of 3000. We could not consider subsamples of smaller sizes because they often yielded singular designs (many components of X are dummy variables taking on positive value with probability $2 - 10\%$). Thus, we dealt with *effectively* a *small sample* despite that $n = 6384$ (see Goldberger (1991) on characterizing close-to-singular designs as, effectively, the small-sample designs).

The first two hypotheses are *decisively rejected*, strongly supporting the conclusion of Koenker and Xiao (2002a). This is notable given that the resampling tests provide accurate and powerful inferences even in effectively small samples, as we saw in Table 2. The rejection of these hypotheses is an additional strong evidence in favor of the quantile inference

paradigm of Lehmann-Doskum,[9] which emphasizes ubiquity of quantile shift effects and the general *impossibility* of describing such treatment effects as merely shifting the location and scale.

The hypothesis of stochastic dominance, the third one, is *decisively supported.* The test statistic is 0, while for rejection it is necessary that it exceeds the value of 2.6. This additional result complements the set of inferences given in Koenker and Xiao (2002a). Thus the bonus offer creates a first order stochastic dominance effect on the unemployment duration, supporting the efficacy of the program.

6. Conclusion

A simple and practical resampling test is offered as an alternative to the Khmaladzation technique, suggested in Koenker and Xiao (2002a). This alternative has optimal power (same power as the test with known critical value) and does not require estimation of non-parametric nuisance functions. It applies both to iid and time series data. Finite-sample experiments provide a strong evidence in favor of this technique and an empirical illustration illustrates its utility.

Appendix A

Proof of Proposition 1 and 2 The result is immediate from A.2-A.3. ■

Proof of Theorem 1 Part II of the proof follows standard arguments for subsampling consistency, as in Politis, Romano, and Wolf (1999). There are few details that we have to fill out before then. We give the proof for the Kolmogorov-Smirnov statistic. Extensions to other statistics defined in the text are straightforward.

I.To prove (i)- (iii), define $\dot{G}_{n,b}(x)$ and write out $\widehat{G}_{n,b}(x)$

$$\widehat{G}_{n,b}(x) = B_n^{-1} \sum_{i \le B_n} 1\Big[\underbrace{\sup_{\tau\in\mathcal{T}} \Big\| \widehat{V}^{1/2}(\tau)\left(\sqrt{b}(v_{n,b,i}(\tau)-g(\tau)) + \sqrt{b}(g(\tau)-v_n(\tau))\right)\Big\| }_{\widehat{A}_i} < x\Big],$$

$$\dot{G}_{n,b}(x) = B_n^{-1} 1\Big[\sum_{i \le B_n} \underbrace{\sup_{\tau\in\mathcal{T}} \Big\| V^{1/2}(\tau)\left(\sqrt{b}(v_{n,b,i}(\tau)-g(\tau))\right)\Big\| }_{A_i} < x\Big],$$

$E\dot{G}_{n,b}(x) = P_n(\mathcal{S}_b \le x)$. For iid case: by $\dot{G}_{n,b}(x)$ being a U-statistic of degree b; and otherwise: by LLN in Politis, Romano, and Wolf (1999), Theorem 3.2.1, combined with contiguity, conclude $\dot{G}_{n,b}(x) \xrightarrow{P_n^*} G(x)$. Next collect two facts: fact 1, uniformly in i

$$\sqrt{\frac{1}{\lambda_n}} \le \frac{\Big\| \widehat{V}^{1/2}(\tau)\left(\sqrt{b}(v_{n,b,i}(\tau)-g(\tau)) + \sqrt{b}(g(\tau)-v_n(\tau))\right)\Big\|}{\Big\| V^{1/2}(\tau)\left(\sqrt{b}(v_{n,b,i}(\tau)-g(\tau)) + \sqrt{b}(g(\tau)-v_n(\tau))\right)\Big\|} \le \sqrt{\lambda_n},$$

[9]The paradigm is formulated in a series of works by Lehmann (1974), Doksum (1974), Koenker and Machado (1999), and Koenker and Bilias (2001).

where $\bar{\lambda}_n = \sup_\tau$ maxeig $\left(V^{-1/2}(\tau)\widehat{V}(\tau)V^{-1/2}(\tau)\right)$, and $\dot{\lambda}_n = \sup_\tau$ maxeig $\left(\widehat{V}^{-1/2}(\tau)V(\tau)\widehat{V}^{-1/2}(\tau)\right)$ by eq-ty 10 on p.460 in Amemiya (1985).[10] Fact 2 follows from Fact 1 and by $\|A\|-\|w\| \leq \|A+w\| \leq \|A\|+\|w\|$,

$$1[A_i < (x/u_n - w_n)] \leq 1[\widehat{A}_i < x] \leq 1[A_i < (x/l_n + w_n)]$$

where $l_n = \sqrt{1/\dot{\lambda}_n}$ and $u_n = \sqrt{\bar{\lambda}_n}$, and w_n is defined below.

By A2 and A3 and assumptions on $\widehat{V}$ and V $w_n \equiv \sup_\tau \sqrt{b}\left\|V^{1/2}(\tau)(v_n(\tau) - g(\tau))\right\| = O_p(\sqrt{b}/\sqrt{T}) \xrightarrow{P_n^*} 0$, $q_n \equiv \max[|u_n - 1|, |l_n - 1|] \xrightarrow{P_n^*} 0$. Thus wp$\to$ 1 $1(E_n) = 1$, where $E_n = \{v_n, q_n \leq \delta\}$ for any $\delta > 0$.

II. Thus for small enough $\epsilon > 0$ there is $\delta > 0$, so that by fact 2: $\dot{G}_{n,b}(x-\epsilon)1(E_n) \leq \widehat{G}_{n,b}(x)1(E_n) \leq \dot{G}_{n,b}(x+\epsilon)1(E_n)$ so that with probability tending to one: $\dot{G}_{n,b}(x-\epsilon) \leq \widehat{G}_{n,b}(x) \leq \dot{G}_{n,b}(x+\epsilon)$. Now pick $\epsilon > 0$ so that $[x-\epsilon, x+\epsilon]$ are continuity points of $G(x)$. For such small enough ϵ, $\dot{G}_{n,b}(x+c) \xrightarrow{P_n^*} G(x-c)$, for $c = \epsilon$ and $c = -\epsilon$, which implies $G(x-\epsilon) - \epsilon \leq \widehat{G}_{n,b}(x) \leq G(x+\epsilon) + \epsilon$ w.p. $\to 1$. Since ϵ and $\delta(\epsilon)$ can be set as small as we like, $G_{n,b}(x) \xrightarrow{P_n^*} G(x)$. Now note that $x = G^{-1}(1-\alpha)$ is a continuity point by assumption. Convergence of quantiles is implied by the convergence of distribution functions at continuity points.

III. (iv) follows from Lifshits (1982) or Davydov, Lifshits, and Smorodina (1998) by A3. ■

Proof of Theorem 2 I. The proof of the first statement is a direct corollary of Theorem 7.3.1 in Politis, Romano, and Wolf (1999). **II.** The class of moment functions f defined prior to the proof is clearly Borel measurable, since these functions are a measurable function of a random vector $v_n(\tau)$. The convergence of subsampling truncated moments thus follows from the definition of ρ_L. **III.** Finally, because $\bar{v}(\tau)$ is non-degenerate uniformly in τ, it has continuous bounded density by Gaussianity, by A.3, $Ev(\tau)^p 1_{K(\tau)}(\bar{v}(\tau))$ can be approximated arbitrarily well by $Ev(\tau)^p f_{K(\tau)}(\bar{v}(\tau))$ for some $v \mapsto v(\tau)^p f_{K(\tau)} \in L(c', k')$ for some c', k'. ■.

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[10]Let λ be the maximum eigenvalue (characteristic root) of the symmetric matrix A. Then $\lambda = \sup_x x'Ax/x'x$.

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TABLE 1. Empirical Rejection Results for 5% level Khmaladze Test

	H= .5 Bofinger			H=.6 Bofinger			H=Bofinger		
	Size	Power		Size	Power		Size	Power	
	$\gamma = 0$	$\gamma = .2$	$\gamma_1 = .5$	$\gamma = 0$	$\gamma = .2$	$\gamma_1 = .5$	$\gamma = 0$	$\gamma = .2$	$\gamma_1 = .5$
$n = 100$	0.101	0.264	0.898	0.035	0.211	0.755	0.016	0.126	0.641
$n = 200$	0.070	0.480	0.988	0.041	0.406	0.990	0.022	0.280	0.964
$n = 300$	0.062	0.622	0.998	0.043	0.665	1.000	0.029	0.416	0.998
$n = 400$	0.043	0.809	1.000	0.043	0.809	1.000	0.035	0.632	1.000

Notes: All results are from Koenker and Xiao (2002b). Symbol H denotes different bandwidth choices relative to the Bofinger rule.

TABLE 2. Empirical rejection results for 5% resampling test (Smirnov Statistic), for various K, $b = K \times n^{2/5}$, using 250 bootstrap draws and 500 Repetitions.

	Subsampling Test (K=5)			Subsampling Test (K=10)			Bootstrap Test (b=n)		
	Size	Power		Size	Power		Size	Power	
	$\gamma = 0$	$\gamma = .2$	$\gamma_1 = .5$	$\gamma = 0$	$\gamma = .2$	$\gamma_1 = .5$	$\gamma = 0$	$\gamma = .2$	$\gamma_1 = .5$
$n = 100$	0.014	0.348	0.980	0.026	0.350	0.954	0.022	0.316	0.968
$n = 200$	0.052	0.752	1.000	0.059	0.728	1.000	0.038	0.728	1.000
$n = 300$	0.058	0.910	1.000	0.058	0.924	1.000	0.074	0.918	1.000
$n = 400$	0.054	0.980	1.000	0.064	0.978	1.000	0.056	0.970	1.000
maximal sim. s.e.	0.009			0.009			0.009		

Notes: All results are reproducible and the programs are available from the author.

TABLE 3. The test results for the re-employment bonus treatment, using $b = 3000$ (subsampling with replacement)

Hypothesis	Null	Alternative	Smirnov Statistic	5% level critical value	Decision
Location-shift	$\delta(\tau) = \delta$	$\delta(\tau) \neq \delta$	2.46	1.31	Reject
Location-scale shift	$\delta(\tau) = \alpha + \gamma\alpha(\tau)$	$\delta(\tau) \neq \alpha + \gamma\alpha(\tau)$	2.47	1.30	Reject
Dominance Effect	$\delta(\tau) \leq 0$	$\exists\tau : \delta(\tau) > 0$	0.00	4.59	Accept

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